LAWS OF THE KINETICS OF DRYING HUMID MATERIALS

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A hypothesis concerning the kinetics of mass exchange during drying is considered. Methods of generalizing the drying curves and drying-rate curves are proposed, and methods of calculating the drying rates and the drying times are outlined. Experimental relationships which describe the heat-exchange kinetics in the case of conductive drying are stated.

An analysis of numerous experimental data on the kinetics of drying of various materials (grain, vegetables, cardboard, chemical compounds, peat, etc.) with various methods (convective drying, conductive drying, combined drying, infrared drying, drying in a boiling layer) used by Soviet and Western researchers led the author to the following hypothesis: "When some material is dried with a particular method and the material has a certain initial moisture content, W_{in}, the quantity $N\tau$ which corresponds to a particular instantaneous moisture content W is constant under all conditions of drying."

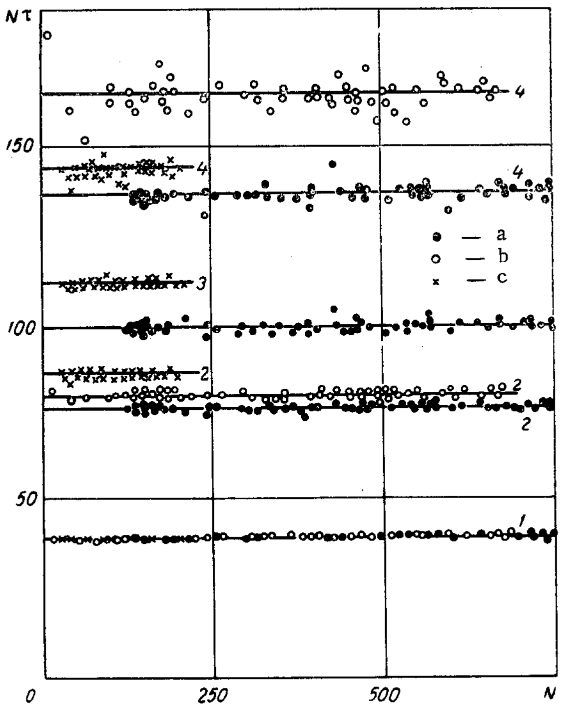

Fig. 1

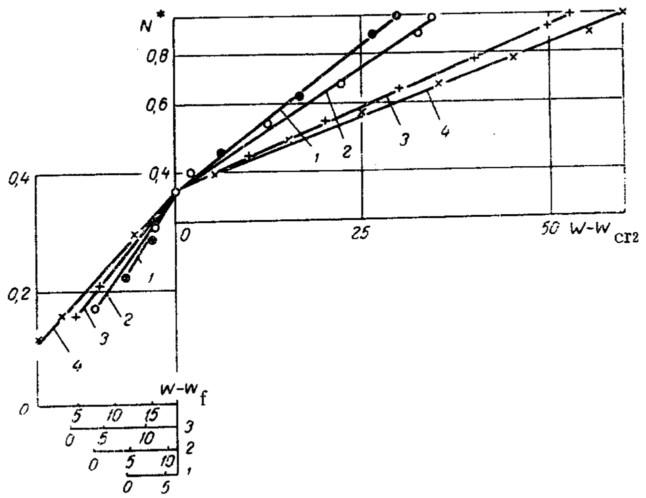

Fig. 2

Fig. 1. Relation between the quantities $N\tau$ (%) and N (%/min) for various W (with W_{in} = 120%): 1) W = 80%; 2) W = 45%; 3) W = 30%; and 4) W = 15%; a) combined drying of 0.1 kg/m^2 cellulose; b) conductive drying of 0.1 kg/m^2 cellulose; c) combined drying of 0.4 kg/m^2 cardboard for roofing.

Fig. 2. Generalized curves of the rate of combined drying of the following types of cellulose: 1) g = 0.05 kg/m^2; 2) g = 0.10 kg/m^2; 3) g = 0.2 kg/m^2; and 4) g = 0.3 kg/m^3.

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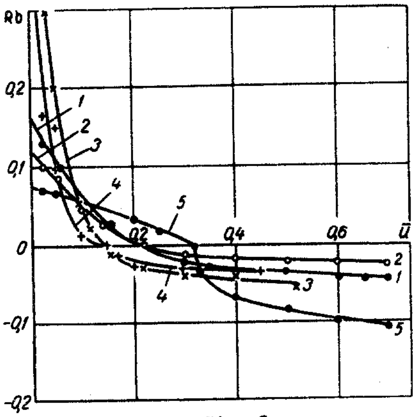

Fig. 3

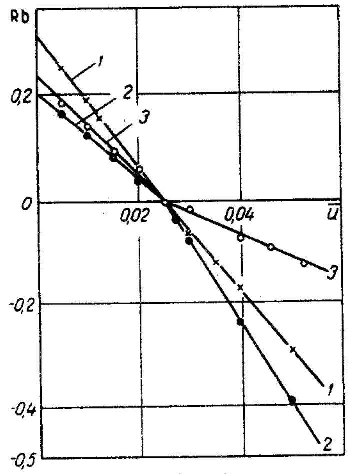

Fig. 4

Fig. 3. Dependence of the Rebinder number Rb in the conductive drying of synthetic cellulose materials (curves 1 and 2) and cellulose (curves 3, 4, and 5) upon ū (kg/kg) at various temperatures of the hot surface, t_{hot}: 1) $t_{hot} = 100°C$; 2) 80; 3) 116 (0.3 kg/m^2); 4) 100 (0.1 kg/m^2); 5) 103 (1.5 mm).

Fig. 4. Dependence of the Rebinder number Rb upon ū (kg/kg) in conductive drying of a 20 mm thick sand layer at various temperatures of the hot surface (t_{hot}) and of ventilating air with velocity 1.5 m/sec (t_c): 1) $t_{hot} = 110°C$, $t_c = 23°C$; 2) $t_{hot} = 90°C$, $t_c = 23°C$; 3) $t_{hot} = 90°C$, $t_c = 90°C$.

The following mathematical formulation of the hypothesis can be used:

$$N_1\tau_1 = N_2\tau_2 = \ldots = N_n\tau_n = (N\tau)_w = \text{const.} \quad (1)$$

The N dependence of $N\tau$, which is shown in Fig. 1 for various W and for conductive and combined (intermittent conductive–convective) drying of 0.1 kg/m^2 cellulose and 0.4 kg/m^2 cardboard corroborates Eq. (1). We infer from the figures that the quantity $N\tau$ remains constant for a particular W when materials of various specific masses are considered, and $N\tau$ is independent of both the drying method and the drying conditions depending only upon W_{in} and W. In the second drying period, a differentiation between the experimental points is observed (each point corresponds to one test). The points are situated close to straight lines which correspond to a particular W value and a certain specific mass of the material to be dried with a particular method. The spread of the experimental points increases slightly with decreasing moisture content but remains within the error limits of the measurements.

This means that all experimental drying curves of a particular material which is dried under various conditions (set of curves) can be converted, at a certain W_{in} value, into a new $W-N\tau$ coordinate system in which the set of curves results in a single curve, the so-called generalized curve of the kinetics of drying [2].

With Eq. (1), a second method of generalizing the drying curves can be derived. This method is based on the fact that the following quantity is constant under any drying conditions of a particular material which is dried with a particular method:

$$\frac{\tau_1}{\tau_{d1}} = \frac{\tau_2}{\tau_{d2}} = \ldots = \frac{\tau_n}{\tau_{dn}} = \left(\frac{\tau}{\tau_d}\right)_w = \text{const.} \quad (2)$$

The set of drying curves, which is characterized by constant W_{in} and W_f, transforms into a generalized drying curve [2] when the set of curves is transferred into a $W-\tau/\tau_d$ coordinate system. The method is particularly suitable when no first stage is observed in the drying process.

A third method of generalizing the drying curves results from the hypothesis: it is based on the fact that when a certain material is dried under any conditions with a particular method, the following quantity is almost constant:

$$K_1\tau_1 \approx K_2\tau_2 \approx \ldots \approx K_n\tau_n = (K\tau)_w = \text{const.} \tag{3}$$

The generalized curve is in this case plotted in $W - K\tau$ coordinates. Generalization is possible when the drying process as a whole occurs in a period of decreasing drying rates. The fact that the quantities $N\tau$, τ/τ_d, and $K\tau$ are invariant during the drying process for a particular W of a material, independent of the drying conditions, is the most general law of the drying kinetics of humid materials, and it can be used, together with the kinetic calculations, when the principles of drying processes are modeled.

The generalized drying curve is constructed from a single test curve obtained by drying a particular material with a certain W_{in} value. The drying curve may have been obtained under any conditions. If the correlation between N (or τ_d or K) and the basic parameters of the conditions is known, the generalized drying curve can be used to recreate the set of drying curves (for a single W_{in} value). The curves which correspond to various conditions of drying a certain material can be recreated without performing experiments.

The generalized drying curve in $W - N\tau$ coordinates can be expressed by three equations (the second drying period comprises two sections) which describe alternately the kinetics of drying [2]. Expressions for determining the drying rates in the two sections of the second period can be obtained by differentiating the equations of the drying kinetics with respect to time.

The actual curve of the drying rate in the second section (the curve was obtained by graphical differentiation of the drying curve) cannot always be replaced by a straight line or, as in our method, by a broken line, because this procedure causes additional errors. It is therefore convenient to use experimental curves of the drying rate and to approximate these curves by empirical relationships.

It follows from the generalized drying curve in $W - N\tau$ coordinates that the relative drying rate N^* ($N^* = dW/d\tau : N$) for a given W is numerically equal to the tangent to the drying curve at the point with W (tangent to the $N\tau$ axis), i.e., N^* can be directly obtained from the generalized drying curve

$$N^* = \frac{1}{N}\frac{dW}{d\tau} = \text{tg}\,(W,\ N\tau) = f(W). \tag{4}$$

Consequently, the quantity N^* is independent of the drying conditions and is only a function of the moisture content for a particular material and a particular drying method. This conclusion is obtained from the method of generalizing drying curves and agrees with the conclusions of G. K. Filonenko concerning the reduced drying rate [3].

The graphical differentiation of a single generalized drying curve, can be performed with rather high accuracy, producing an $N^* - W$ curve, the so-called generalized curve of the drying rate. This curve consists of several sections and comprises a set of drying-rate curves corresponding to various conditions.

An analysis of the generalized drying-rate curves of various materials revealed that the majority of curves in the second period of the drying process can be represented by two conjugated sections or by an exponential curve or by both an exponential curve and a straight line. These possibilities comprise almost all forms of the drying-rate curves [1].

It follows from the generalized drying-rate curves of Fig. 2 that the curve can be represented by a broken line which is inflected at a second critical moisture content W_{cr2}. The function N^* changes in the transition from the first curve section to the second, which means that both kinetics and dynamics of drying are different for the various sections of the second drying stage.

The quantities N_1^* in the first section and N_2^* in the second section of the second drying stage are calculated with the following empirical relationships in our case (see Fig. 2):

$$N_1^* = a_0' \exp\,[a_1'(W - W_{cr2})], \tag{5}$$

$$N_2^* = a_0'' + a_1''(W - W_f). \tag{6}$$

The coefficients of Eqs. (5) and (6) can be easily obtained from the generalized curve of the drying rate in semilogarithmic or nonlogarithmic coordinates. But we emphasize that only one experimentally obtained curve of the drying kinetics is needed for this purpose, the curve being recorded under some fixed conditions. The coefficients depend upon the specific mass, the material, and the drying method employed.

The drying rate at any time of the second stage can be determined from the quantity N in the first stage for any drying conditions, for any N*, and for any moisture content (or drying time) by the formula

$$\left|\frac{dW}{d\tau}\right| = N^*N. \tag{7}$$

Studies of the drying process and the evaluation of experimental data with generalized drying curves and drying-rate curves made it possible to switch from a single experiment to quantitative results for a large number of cases corresponding to various drying conditions. No additional experiments need be made for this transition. This reduces both the time and the means required for experimental investigations of the drying process of a particular material.

The proposed analysis of the drying process and the generalization of experimental data are particularly useful in branches of industry and national economy in which very dissimilar materials and products are dried, the materials differing in form and size, moisture adhesion, and their mechanical properties and structures. Moreover, the generalization methods can be used when the dynamics of drying is investigated, particularly when fields containing moisture are analyzed [4].

Our analysis of the kinetics of mass-exchange drying permitted us to propose four methods of calculating the drying time. The first method is based upon kinetic equations which alternately describe the kinetics of the drying process. In this method, the duration of the drying process is given by the equation

$$\tau_d = \frac{1}{N}\left(W_{in} - W_{cr1} + \frac{1}{\chi_1}\lg\frac{W_{cr1} - W_{eq}}{W_{cr2} - W_{eq}} + \frac{1}{\chi_2}\lg\frac{W_{cr2} - W_{eq}}{W_f - W_{eq}}\right). \tag{8}$$

The first calculation method, which is an extension of A. V. Lykov's method, is very accurate, as could be shown by test calculations. The quantities W_{cr1} and W_{cr2} and the relative coefficients χ_1 and χ_2 of the drying process are obtained from the generalized drying curve in semilogarithmic coordinates.

The second method is based upon immediate application of the generalized drying curve in $W-N\tau$ coordinates. The computation formula for the determination of the duration of the drying process between some value W_{in} and a given value W has the form

$$\tau_d = \frac{1}{N}(W_{in} + W') = \frac{1}{N}[W_{in} + (N\tau)_w - W'_{in}]. \tag{9}$$

Five quantities appear in this formula; two of these quantities (W_{in} and W) are given, N can be calculated from known relationships, and W'_{in} (the initial moisture content for which the generalized drying curve was constructed) and $(N\tau)_W$ are obtained from the generalized drying curve. Compared with the first calculation method, the second method is more convenient and much simpler for engineering application with the same accuracy as the first method.

The third method is based on the generalized drying curve in $W-\tau/\tau_d$ coordinates. The duration of the process between W'_{in} and W'_f (the curve was plotted for these moisture contents) is determined from the generalized curve with the aid of a single experimental value of W and τ or from an N value, provided that N is known. In the latter case, by assuming some W value in the first stage, the time τ of drying from W'_{in} to W can be determined, i.e., the duration of the drying process can be determined from W and τ. The computation formula has the form

$$\tau_d = \frac{\tau}{\left(\frac{\tau}{\tau_d}\right)_w}. \tag{10}$$

The quantity $(\tau/\tau_d)_W$ in Eq. (10) is taken from the generalized drying curve obtained from experimental W and τ values.

The fourth method is based upon the generalized drying curve in $W - K\tau$ coordinates. The duration of the drying process is calculated from the formula

$$\tau_d = \frac{(K\tau)_w}{K}. \tag{11}$$

The quantity $(K\tau)_W$ is obtained from the generalized drying curve for a given W to which the drying process is conducted.

It is generally accepted [1] that a knowledge of the dependence of the Rebinder number Rb, and of the temperature coefficient b of drying, on the moisture content is necessary for calculations of the basic parameters of the heat-exchange kinetics (density of heat flow and average temperature of the material). These dependencies can be obtained from experiments or from analytic solutions of the differential equations of heat exchange and mass exchange.

The density $q(\tau)$ of the heat flow at any time of the drying process can be determined from the basic equation of the drying kinetics [1]. When Eqs. (5) and (6) are used, the basic equation of the drying kinetics assumes the form

$$\begin{aligned} q(\tau) &= qa_0 \exp[a_1(W - W_1)][1 + \mathrm{Rb}] \\ &= gr\frac{N}{100} a_0 \exp[a_1(W - W_1)][1 + \mathrm{Rb}]. \end{aligned} \tag{12}$$

The evaluation of experimental results has shown that the kinetics of the heat exchange in the second stage depends upon two temperature coefficients of drying and two Rebinder numbers which are characteristic of the first and second sections of the second stage. The dependence of these quantities upon the moisture content differ in the two sections of the drying stage. In the case of conductive drying, the values of b and Rb differ also in their signs (b_1 and Rb_1 are negative and b_2 and Rb_2 are positive).

Experimental values of the coefficients b in conductive drying of various materials (synthetic cellulose, clay, sand, and glass fibers) can be approximated by the following functions:

a) in the first section of the second stage:

$$b_1 = b_{01}(\bar{u} - \bar{u}_{cr2}) \tag{13}$$

or

$$b_1 = \mathrm{const}; \tag{14}$$

b) in the second section of the second stage:

$$b_2 = b_{02}(\bar{u}_{cr2} - \bar{u})^{n''}. \tag{15}$$

The quantity b_{01} is always much smaller than b_{02}; the n'' value varies between 1 (sand, clay) and 3.13 (glass fibers). The quantities b_{01} and b_{02} increase slightly with increasing temperature of the hot surface and, in addition, b_{02} depends upon the g value of the material. The exponent in Eq. (15) depends only upon the thickness of a particular material and increases with decreasing thickness. Consequently, the quantities b_1 and b_2 depend slightly upon the parameters of the drying conditions.

The functions $\mathrm{Rb} = f(\bar{u})$ of capillary-porous colloidal materials (synthetic cellulose materials and cellulose) have the following forms in the first and second sections of the second stage:

$$\mathrm{Rb}_1 = D_1(\bar{u} - \bar{u}_{cr2})^{n_1}, \tag{16}$$

$$\mathrm{Rb}_2 = D_2(\bar{u}_{cr2} - \bar{u})^{n_2}. \tag{17}$$

The evaluation of experimental data has shown that the quantity n_1 is independent of the conditions and of the g value of the material but depends on the type of the material ($n_1 = 0.25$ for cellulose and $n_1 = 0.38$ for synthetic cellulose materials). The exponent n_2 is independent of t_{hot} and decreases with increasing g ($n_2 = 1.69$ for synthetic cellulose materials). The quantities D_1 and D_2 depend upon t_{hot} and g.

The functions Rb = f($\bar{u}$) for capillary-porous bodies (sand, glass fibers) are

$$\mathrm{Rb}_1 = D_1' (\bar{u} - \bar{u}_{cr2}), \tag{18}$$

$$\mathrm{Rb}_2 = D_2' (\bar{u}_{cr2} - \bar{u}), \tag{19}$$

where D_1' and D_2' depend upon the drying conditions.

Differences in moisture adhesion to the materials are responsible for difference between the dependencies of the Rb numbers upon the moisture content of capillary-porous colloidal bodies and capillary-porous bodies.

The quantitative and qualitative changes of the Rb numbers during conductive drying of capillary-porous colloidal materials and capillary-porous materials are shown in Figs. 3 and 4, respectively. The curves were constructed from equations for the corresponding Rb values; the points correspond to experimental data. It follows from the figures that Rb increases with increasing $\bar{u}$. The increase is more significant in the case of capillary-porous colloidal bodies than in the case of capillary-porous bodies. The magnitude of Rb depends to some extent on the parameters of the drying conditions. For example, when the temperature of the air blowing over a sand surface increases, the absolute value of Rb_1 decreases considerably (Fig. 4), whereas the absolute value of Rb_2 increases, but less rapidly than the decreasing Rb_1.

The Rb_2 values prove that a conductive drying process must be stopped at the desired final moisture content. Apart from changes in the technological properties of a material, overdrying results in a considerable overconsumption of energy because Rb_2 increases sharply with decreasing $\bar{u}$.

The basic equation of the drying kinetics with proper account for the sign of the Rb number has the form:

for the first section of the second stage of conductive drying

$$q_1^* = N_1^* (1 - \mathrm{Rb}_1), \tag{20}$$

for the second section

$$q_2^* = N_2^* (1 + \mathrm{Rb}_2). \tag{21}$$

When at the beginning of the first section of the second stage, the density of the heat supplied to the material amounts to 95% of the density of the heat flow spent for evaporation ($\mathrm{Rb}_1 = 0.05$), 5% of the heat spent for evaporation is taken directly from the material, i.e., from the heat accumulated by the material. Consequently, the temperature of the material decreases. For $\mathrm{Rb}_2 = 0.13$, the material absorbs heat which is spent for evaporation and heating. In this case, 13% of the heat spent for evaporation is lost by heating the material.

Equation (20) and (21) and the relationships Rb = f($\bar{u}$) make it possible to determine, at any time of the process, the density of the heat flux supplied to a material even when the heat exchange is complicated. This calculation of $q(\tau)$ made it possible to work without the heat-exchange coefficient $\alpha(\tau)$ and to formulate boundary-value problems of drying without introducing boundary conditions of the third kind, but to replace the boundary conditions by conditions of the second kind.

Calculations of the average temperatures of a material at any time of the second drying stage are in good agreement with the experimental results when b is constant or varies according to a linear law.

The proposed methods of calculating and analyzing the kinetics of heat exchange and mass exchange during the drying process of moist materials could be directly used in investigations of the drying process and in practical applications, in the automation of the drying process, and in calculation of drying apparatus. The methods can also be successfully employed in an analytic theory of moisture and heat transfer in materials which are being dried.

NOTATION

W	the moisture content (%);
$\tau_1, \tau_2, \ldots, \tau_n$	the drying time during which the moisture content varied between W_{in} and W;
$N_1, N_2, \ldots, N_n$	the drying rates in the first stage under various conditions (%/sec);

$\tau_{d1}, \tau_{d2}, \ldots, \tau_{dn}$	the duration of the drying process from constant W_{in} to constant W_f under various conditions;
$K_1, K_2, \ldots, K_n$	drying coefficients under various conditions;
g	the density of absolutely dry material, kg/m^2;
r	the specific heat of evaporation, kcal/kg;
$b = d\bar{t}/d\bar{u}$	the temperature coefficient of drying, degree/kg/kg;
$\bar{u}$	the integral moisture content, kg/kg;

Subscripts

in	initial;
f	final;
cr_1	first critical;
cr_2	second critical;
eq	equilibrium.

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